3D modeling of magneto-elastic behavior using simplified multi-scale model : application on power transformer core

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Recently, a 2D magneto-mechanical coupled approach based on a simplified multi-scale modeling (SMSM) is proposed to the vibration prediction of power transformer core. An sequential approach from magnetic resolution and mechanical resolution is considered using the SMSM which describes both magnetic and magnetostrictive behavior. To better study the magneto-elastic effect on power transformer core, we extend its results to three dimensions in this work. Core distortion and vibration are finally shown under current excitations.

***Index Terms*—Magnetostriction, transformers, vibration, multi-scale modeling, finite element method.**

I. Introduction

WITH THE raising power density of electrical devices such as electrical machine and power transformer, ferromagnetic material tends to operate under high magnetic flux. This induces a significant increasement of magnetostriction and magnetic force, leading to the strong mechanical vibration and audible noise. It is really difficult to understand these vibrations by measurements, due to the various of factors non-controllable such as assembly precision, material non-homogeneity and unbalanced mass. These factors make experimental measurement non-repeatable from prototype to prototype. Therefor, numerical simulation becomes important for the design of these devices and for better understanding of the source of noise. Commercial modeling tool is still limited to its low computing efficiency and a lack of choice of material model. Recently, a 2D modeling chain is proposed based on a simplified multi-scale model (SMSM). Comparisons of numerical simulation and experimental measurements are made, with good overall precision [1]. However, 2D modeling is still limit to show for example the out-of-plan vibration. Some vibration modes can only be obtained by 3D modeling. This work is the extension of previous model from 2D to 3D. The implementation of Freefem++ is invoked [2]. An embedded 'non-cut' tri-phase power transformer with no air-gap is used as example, leading to 3D vibration estimation of the power transformer core.

II. Numerical models

A. Magnetic parts

For a given coil excitation $\mathbf{J}_\mathrm{S}$, let us denote $\mathbf{B}$, $\mathbf{H}$ respectively the magnetic flux density and magnetic filed, the magnetic problem reads :

$$\mathrm{div}\mathbf{B} = 0;\ \mathbf{rotH} = \mathbf{J}_\mathrm{S};\ \mathbf{B} = \mu_0(\mathbf{H} + M), \tag{1}$$

where μ_0 represents the vacuum permeability and M is the magnetization. Using the Ω scalar potential which verifies

$$\mathbf{H} = \mathbf{H}_\mathrm{S} - \mathbf{grad}\Omega,$$

where $\mathbf{H}_\mathrm{S}$ is reconstructed such that $\mathbf{rotH}_\mathrm{S} = \mathbf{J}_\mathrm{S}$, we can obtain

$$\mathrm{div}(\mu_0(\mathbf{H}_\mathrm{S} + \mathbf{M} - \mathbf{grad}\Omega)) = 0. \tag{2}$$

The term $\mathbf{M}$ is coupled with the following SMSM and mechanical part.

B. Mechanical parts

Let us denote the $\mathbf{u}$ the displacement vector, the mechanical problem reads:

$$\mathrm{div}(\mathbb{C} : \epsilon) - \rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \mathrm{div}(\mathbb{C} : \epsilon_\mu), \tag{3}$$

where $\mathbb{C}$ is the stiffness tensor of the medium, $\epsilon = (\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ presents the strain-displacement and ρ is the mass density. The free magnetostrictive strain ϵ_μ is computed from the SMSM part and then transformed into an equivalent force density as a classical body force.

C. Constitutive law

A simplified version of a full multi-scale magneto-mechanical model (MSM) [3], [4] is considered as in 2D case [1]. This model allows an accurate modeling of anhysteretic magnetic and magnetostrictive behaviors of ferro/ferrimagnetic materials, and takes the effect of multiaxial mechanical stress into account. The number of internal variables of such model is nevertheless too high to be implemented in a complex structure model with a high number of degrees of freedom. The simplified version (SMSM) where the magnetic material is considered as an equivalent single-crystal (including anisotropy effects) has been recently proposed for that purpose [5]. This single-cristal consisting of magnetic domains is oriented to different directions in 3D space. Local free energy of a magnetic domain (α) oriented in direction ($\vec{\gamma}_\alpha$) is expressed as the sum of three contributions if the stress effect is neglected (weak coupling):

$$W^\alpha_{tot} = W^\alpha_{mag} + W^\alpha_{an} + W^\alpha_{conf} \tag{4}$$

W^{α}_{mag} is the Zeeman energy, introducing the effect of the applied magnetic field on the equilibrium state. W^{α}_{an} is an anisotropic energy term related to the crystallographic texture and the magneto crystalline anisotropy. W^{α}_{conf} is a configuration energy term, which allows some peculiar initial distribution of domains (residual stress effect, demagnetizing surface effect...). A Boltzmann like function is used to calculate the volume fraction of domains in different directions f_{α}.

$$f_{\alpha} = \frac{exp\left(-A_s W^{\alpha}_{tot}\right)}{\sum\limits_{\alpha} exp\left(-A_s W^{\alpha}_{tot}\right)} \tag{5}$$

where A_s is an adjusting parameter. Macroscopic quantities are finally obtained by averaging the microscopic quantities over the single crystal volume (6)(7).

$$\mathbf{M} = \sum_{\alpha} f_{\alpha} \mathbf{M}^{\alpha} \tag{6}$$

$$\epsilon_{\mu} = \sum_{\alpha} f_{\alpha} \epsilon^{\alpha}_{\mu} \tag{7}$$

From a given magnetic field $\mathbf{H}$, the SMSM gives then the corresponding free magnetostriction strain ϵ_{μ} and magnetization M.

III. Numerical application

In the numerical part, a three-phase power transformer made of Non-oriented (NO) FeSi is considered as shown in Fig. 1 , which is only excited by a central coil carrying a sinusoidal flux. This leads to a maximum induction of B = 1.2T.

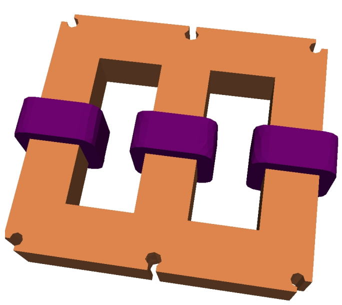
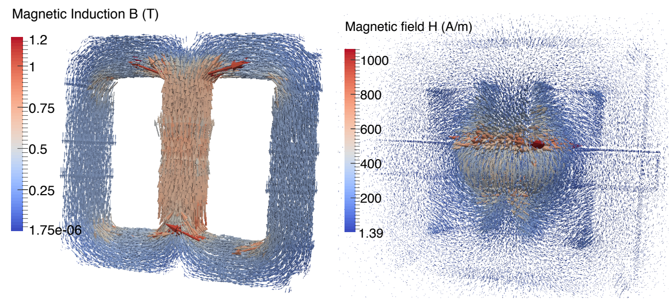


Fig. 1. Geometry of studied transformer.

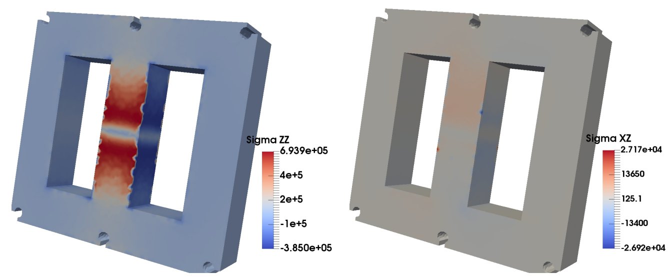
The distribution of magnetic flux density $\mathbf{B}$ and magnetic filed $\mathbf{H}$ obtained by the magnetic parts is shown in figure 2. Only with the central coil excitation, the magnetic flux density which is concentrated in the transformer core and the magnetic field which is around the coil are observed, verifying a correct computation for the magnetic part.

Fig. 3 displays the different components of equivalent magnetostrictive stress tensor $\sigma_{\mu} = \mathbb{C} : \epsilon_{\mu}$, namely $\sigma_{\mu,\text{zz}}$, $\sigma_{\mu,\text{yz}}$ and $\sigma_{\mu,\text{xz}}$ which could not be considered in 2D case. It can be seen that the magnetostriction induced stress is not negligible in the direction z. More analysis and comparisons will be completed in the full paper.

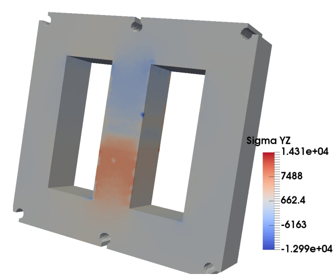


(a) Magnetic flux density $\mathbf{B}$ (b) Magnetic field $\mathbf{H}$

Fig. 2. Distributions of the magnetic flux density and field in the transformer.



(a) $\sigma_{\mu,\text{zz}}$ (b) $\sigma_{\mu,\text{xz}}$



(c) $\sigma_{\mu,\text{yz}}$

Fig. 3. Spatial distributions of different component of stress tensor σ.

IV. Conclusion

To well understand the vibration and audible noise of power transformer, it is necessary to study the magneto-elastic coupled problem due to the magnetostrictive effect. A 3D extension work is carried out based on our 2D modeling chain code and SMSM, leading to a better understanding of the different vibration mode of power transformer. More analysis will be presented in the full paper.

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